A derivation of the generalized Korf growth equation and its application

LI Feng-ri (李凤日)*
(Northeast Forestry University, Harbin 150040, P. R. China)

ZHAO Bao-dong (赵宝东)
(Muling Forestry Bureau of Heilongjiang Province, Mudanjiang 157513, P. R. China)

SU Gui-lin (苏贵林)
(Hongqi Forest Farm of Daqing City, Daqing 163813, P. R. China)

Abstract: Based on the biological hypothesis of tree growth, the generalized Korf growth equation, was derived theoretically. From a standpoint of applications, the equation can be used in two ways associated with the power exponent of p, and two types of growth equations: the Korf-A (p> 1) and the Korf-B (0< p <1) were developed and between them, there is the Gompertz equation (p = 1) to separate each other. All of the three types of equations are independent. It was concluded that the Korf-A equation could be used to describe the growth of trees, of which inflection point is between 0 and A/e, while the Korf-B equation with the inflection point between A/e and A could be applied to describe the biological population growth. It was found that the Korf-A equation had a better property in describing the growth process of a tree or a stand and its applications to predicting height growth and stand self-thinning showed general good fitness.

Key words: Korf equation; Growth model; Self-thinning; Model fitting
CLC number: S71 **Document code:** A **Article ID:** 1007-662X(2000)02-0081-08

Introduction

Five growth equations among great many of plant growth models are often used in describing the biological process of plants or populations over time (in Table 1).

The logistic equation is used probably the one most frequently used in simulating the population dynamics in ecology. In this equation, the relative growth rate of plants is expressed as a declining linear function of the size and inflection point of its curve corresponds to one-half of final size (y_{max}). With the characteristics, the equation was often used in predicting biological population. However, it was found that the equation was the least accurate among 5 sigmoid equations (see Table 1) in predicting tree growth (Osumi & Ishikawa 1983; Zeide 1989), because the relative growth rate ($\frac{1}{y}\frac{dy}{dt}$) of a tree forms a concave curve rather than a linear curve over y.

The Mitscherlich equation was used to describe the response of plants to their environment. This equation was known as the *law of diminishing returns* in agriculture and economics and as the *law of mass action* in chemistry. The original equation form proposed by Mitscherlich was:

$$y = A\left(1-e^{b_1 x_1}\right)\left(1-e^{b_2 x_2}\right)\cdots\cdots \quad (1)$$

Where y is yield, $x_1, x_2\cdots\cdots$ is growth control factors.

Considering time as the most important factor for plant growth, the equation (1) is the simplest among the five growth equations in Table 1, which does not have inflection point. It presents unrealistic picture of tree growth, because there is an inflection point in growth case of tree.

The Gompertz (1825) equation was designed to describe age distribution in human population (Zeide 1993). A century later it was applied as a growth model. It was derived from the assumption that the relative growth rate was proportional to the logarithm of attenuate factor (y_{max}/y). Obviously, the Gompertz equation was based on the individual rather than a population dynamic. The inflection point occurs when current size is equal to y_{max}/e, that is about one-third of the final size. Many researchers found that the

Biography: *LI Feng-ri (1963-), male, professor of Northeast Forestry University, Harbin 150040, P. R. China.
Received date: 2000-01-15
Responsible editor: Song Funan

Gompertz's equation was appropriate to use in biology as well as in forestry. However, Zeide (1989) found that the Gompertz's equation was less accurate than the Korf and the Chapman-Richards equation.

In forestry, the Chapman-Richards equation has been used more than any other functions for its flexibility (Osumi & Ishikawa 1983; Zeide 1993; Li 1995). The Chapman-Richards equation was derived from Bertalanffy equation "When limitations imposed by its theoretical background are discarded" (Richards 1959). The difference between two equations is that the parameter *m,* restricted to a value of three in Bertalanffy's case, can assume any value of m>0 in the Chapman-Richards' equation. The theoretical growth equations of the Mitscherlich, the Gompertz and the logistic are special cases for m=0, $m\to 1$, and m=2, respectively. The Chapman-Richards equation has often been used to develop tree or stand growth models, for examples, growth models of basal area (Pienaar & Turnbull 1973), models of height growth or site index curves (Brewell *et al.* 1985), tree growth analysis (Osumi & Ishikawa 1983), and self-thinning equation (Naito 1984).

The Korf equation were proposed by Korf in 1939 (Zarnovican 1979), who was Czechoslovak forest researcher, and has been rediscovered several times, in particular by Lundqvist (1957). Lundqvist developed a model of height growth by using the Korf equation for plantation of pine and spruce. After then, some researchers have successfully applied the Korf equation to describe the height and diameter growth (Stage 1963; Zarnovican 1979; Brewer *et al.* 1985; Li 1987,1995; Zeide 1989).

Depending on the biological hypothesis of tree growth, this paper is intended to derive a generalized Korf equation theoretically and analyze suitability of the equation for use. Biological meanings of the parameters and the properties of the equation describing the characteristics of tree or stand growth will be also discussed.

Structure and analysis of theoretical equations

Five theoretical equations listed in Table 1 can be expressed in following two forms (Zeide, 1989, 1993):

$$\ln(y') = k + p\ln(y) - q\ln(t)$$

or

$$y' = k_1 y^p t^{-q} \quad (2)$$

$$\ln(y') = k + p\ln(y) - qt$$

or

$$y' = k_1 y^p e^{-qt} \quad (3)$$

Where $y' = \frac{dy}{dt}, \quad k > 0, \quad p > 0, \quad q > 0, \quad k_1 = e^k$

The equation (2) is referred to as the PD (the power decline of age) form and the equation (3) is called as the ED (the exponential decline of age) form. The common feature in both forms is that growth expansion is proportional to current tree size, *y*, raised to some positive power, *p*. Both forms differ in the way the decline component of growth is associated with age (*t*). It is expressed by a power function of *t* for PD form (e.g. Korf equation) and an exponential function of *t* for ED form (e.g. the logistic, Gompertz, Mitscherlich, and Chapman-Richards equations) (see Table 1)

Table 1. Theoretical growth equations and its components*

Equation name	Expression	Theoretical assumption	Components: Expansion	Components: Decline
Logistic	$y = \frac{A}{1+ce^{-bt}}$	$\frac{dy}{dt} = by(1-\frac{y}{A})$	$\frac{bc}{A}y^2$	e^{-bt}
Mitscherlich	$y = A(1-ce^{-bt})$	$\frac{dy}{dt} = b(A-y)$	Abc	e^{-bt}
Gompertz	$y = Ae^{-be^{-ct}}$	$\frac{dy}{dt} = cy(\ln A - \ln y)$	bcy	e^{-ct}
Chapman-Richards	$y = A(1-e^{-bt})^c$	$\frac{dy}{dt} = \alpha y^m - \beta$	$A^{\frac{1}{c}}bcy^{\frac{c-1}{c}}$	e^{-bt}
Korf	$y = Ae^{-bt^{-c}}$	$\frac{dy}{dt} = qy(\ln A - \ln y)^p$	bcy	$t^{-(c+1)}$

● **Where** y = tree or stand size; t = age; A, b, and c = parameters of equations

In theoretical growth equations of the ED forms, the Chapman-Richards equation covering all the

other three equations is mainly analyzed in this paper. According to the parameter m, the Chapman-Richards function was divided into three growth types by Osumi and Ishikawa (1983)(see Table 2)

Table 2. Growth types of the Chapman-Richards growth equation

m value	Growth type	Inflection point
$0 \le m < 1$	Mitscherlich	$y_I = \in [0, A/e]$
$m \to 1$	Gompertz equation	$y_I = A/e$
$m = 2$	Logistic	$y_I = \in [A/e, A]$

They concluded: "in generally, the life-long of a tree belongs to the Mitscherlich's type ($0 \le m < 1$), while the seasonal growth of an organ of a tree belongs to the logistic type ($m > 1$)." It was wrong that the logistic equation was indifferently used to describe the growth of whole mass or stem mass of a tree in the past. This was also demonstrated by other researchers that the logistic equation is not suitable for describing the growth of a tree, but for simulating the population dynamics or growth of an organ. Therefore, the Chapman-Richards equation is effective for modeling tree growth, only when $0 \le m < 1$.

However, the Chapman-Richards equation has been referred to as the most flexible growth equation and applied widely in modeling tree and stand growth. It is because the assumptions, background of derivation, area of applications, and biological capacities of the parameters for the Chapman-Richards function haven't been profoundly analyzed when it was used in forestry. In practical applications of the Chapman-Richards equation, unusual cases appeared occasionally, in which the parameter m was less than zero ($m < 0$). In such cases, the Chapman-Richards equation is failed to coincide with the assumption. Otherwise, the capability of analysis is not as good as expected. When the estimated parameter m is very small, the value of inflection point reveals quite different from observed one of the tree growth. In addition, the inflection point doesn't exist for case $m < 0$. These don't coincide with the fact of tree and stand growth (Li 1995). The reasons are: 1). The basic hypothesis of the Chapman-Richards equation was based on surface rule of animal growth and not suitable for tree growth especially height growth. 2) In assumption, the upper bound was not restrained and the estimated value of asymptotic parameter A depended on the other parameter (for example m). 3) The decline component of tree growth was not a power function of age. Zeide (1989, 1993) found that the relative growth rate of tree was an exponential function rather than a power function of age.

The Korf equation as the PD form reveals to more suitable to describe tree growth. Although this equation wasn't known well in the past, researches found that the Korf equation was substantially more accuracy than the Chapman-Richards function recently (Brewer *et al.* 1985; Li 1987; Zeide 1989). Using average growth of thousands of stem analysis of different species from different locations, it was shown that the Korf equation's standard error of estimate was 2.1, 3.4, and 4.8 times less than the Chapman-Richards, Gompertz, and Logistic equations, respectively. Otherwise, the capability of analysis and flexibility of the Korf equation were better than the Chapman-Richards equation (Li 1995).

Derivation of the generalized Korf growth equation

Theoretical derivation

Biological growth results from the interaction constructive metabolism (anabolism) and destructive metabolism (catabolism). Associated with biotic potential, photosynthesis, absorption of nutrients, etc, the anabolism belongs to positive component of biological growth and represents the innate tendency toward exponential multiplication. While the catabolism embodies the restraints imposed by external (competition, limited resources and stress) and internal (aging and self-regulatory) factors, as well as their combination (respiration). Therefore, postulates of growth equation are often formulated in pairs that reflect the multiplicative and limiting components imposed by finite space. In general, two postulates include:

1) Because of biological characteristics, growth is multiplicative;

2) Because the environmental resistance, the relative growth rate is always decreasing (Minots law).

Here y represents tree size at time t. $\frac{dy}{dt}$ is growth rate and $\frac{1}{y}\frac{dy}{dt}$ represents relative growth rate. Because of the physiological characteristics of tree growth, regardless of environmental limit following two postulates of tree growth exist. 1) y increases with time, t, and an upper limit ($y_{\max}$) exists. Let $A = y_{\max}$; 2) As y increasing, a tree increases in the respiration and decreases in the activity. As a results, it bring out self-restraints and the effect becomes severe when the environmental resistant imposed. These two assumptions of tree growth can be formulated using a general "self-control model":

$$\frac{1}{y}\frac{dy}{dt} = g(y) \tag{4}$$

1) In equation (4), $g(y)$ should satisfy: For

$y \in [0, A]$, $g(y)$ forms a continuous function and greater than or equal to 0.

2) When $y \in [0, A]$, $\frac{dg(y)}{dt} \le 0$. This represents that the growth rate decreases with y increasing (self-restraints).

The typical sigmoid growth equation meets the conditions of "self-control model". For tree growth, There are two equilibrium points, $y = 0$ and $y = A$ in equation (4). The equilibrium point, $y = 0$, is unstable, but the other one, $y = A$, possesses asymptotic stability.

Here, A/y represents attenuate factor of relative growth rate and it gradually decreases as y increases. This procedure indicates that, as the level of $\frac{1}{y}\frac{dy}{dt}$ attenuates as y increasing, $\frac{1}{y}\frac{dy}{dt}$ decreases with A/y decreasing. After analyzing growth procedure of a tree, it was found that $g(y)$ was a non-linear function. With the constraint $\frac{1}{y}\frac{dy}{dt} \to 0$ as $y \to A$, the hypothesis of a tree growth is expressed as (see Figure 1):

$$\frac{1}{y}\frac{dy}{dt} = g(y) = q(\ln\frac{A}{y})^p \qquad q, p > 0 \tag{5}$$

where q is innate growth rate, p is power exponent of the attenuate equation, A is asymptotic size.

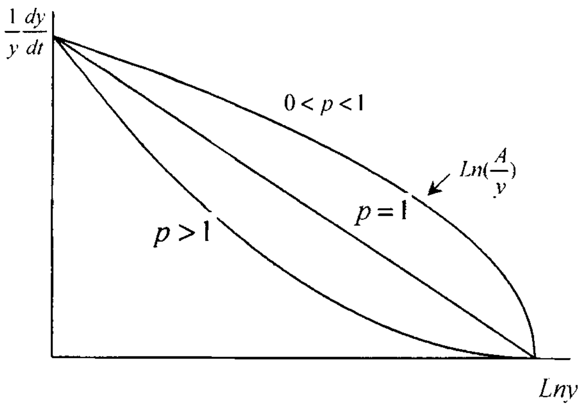

Fig.1. Attenuate procedure of tree growth

The equation (5) is called as an attenuate equation of tree growth. When q is identical, the higher the p value is, the faster the attenuate speed of relative growth rate is. In this case, the action of self-restrain increases and the growth rate decreases. When parameter p is identical, the higher the q value is, the faster the tree growth is. Obviously, the attenuate equation (5) is a special expression of "the self-control model" in ecology. Equation (5) can be rewritten as:

$$\frac{dy}{dt} = qy(\ln A - \ln y)^p \tag{6}$$

Integral form of the differential equation (6) is

$$y = A\exp\left\{-[(p-1)q + c']^{-\frac{1}{p-1}}\right\} \tag{7}$$

Where c' is integral constant. With the initial condition that $y(t) = y_0$ when $t = 0$, equation (7) becomes:

$$y = A\exp\left[-b(c \pm t)^{-\frac{1}{p-1}}\right] \quad A,\ b>0 \text{ and } c\ge 0 \tag{8}$$

Where $b = [\pm(p-1)q]^{-\frac{1}{p-1}}$, $c = \left(\frac{1}{b}\ln\frac{A}{y_0}\right)^{-(p-1)}$,

when $0 \le p < 1$,"+" and $p > 1$, "–"

The equation (8) is called as the generalized Korf growth equation. According to p value in the equation (8), two kinds of equations were divided:

1) When $p>1$ and $q>0$, from equation (8) we obtain

$$y = A\exp\left[-b(t + c)^{-\frac{1}{p-1}}\right] \quad A,\ b>0 \text{ and } c\ge 0 \tag{9}$$

2) With $0 \le p < 1$ and $q>0$, we get

$$y = A\exp\left[-b(c - t)^{\frac{1}{1-p}}\right] \quad A,\ b>0 \text{ and } c \ge t_{\max} \tag{10}$$

The equation (11) and equation (12) are called as Korf-A equation and Korf-B equation, respectively. Otherwise, with p=1, the following expression could be obtained from the attenuate equation (5):

$$\frac{1}{y}\frac{dy}{dt} = q(\ln A - \ln y) \tag{11}$$

From equation (11), the Gompertz growth equation could be derived.

The generalized Korf equation is regarded as a general growth function and composed of two parts, the range of the Korf-A type (p>1) and that of the

Korf-B type ($0 \le p < 1$). Between them, there is the "wall" of the Gompertz equation (p=1) to separate each other. The approach of dividing growth type is similar to Osumi and Ishikawa (1983) (see Table 2).

Properties of the Korf-A equation

The Korf-A equation includes four parameters, some of them are independent of each other (such as A and p), but the others are combined parameters (such as b and c). It has following properties:

1) There is two asymptotic lines: as $t \to 0^+, y \to y_0$ and as $t \to +\infty, y \to A$.

2) y is increasing function related to t, for p>1

$$\frac{dy}{dt} = y\frac{b}{(p-1)}(t+c)^{-\frac{p}{p-1}} > 0$$

3) The equation has an inflection point at

$$t_I = \left(\frac{b}{p}\right)^{p-1} - c, \quad y_I = Ae^{-p}$$

Because of p>1, inflection point of the Korf-A equation is $y_I \in [0, A/e]$. Curve appears typical sigmoid and non-symmetric for $t \ge 0$. The Korf-A growth function can be used to describe sigmoid curves of which the inflection point is between 0 to A/e. As $p \to 1$, the Korf-A equation is approached to the Gompertz function. Tree's growth exactly belongs to sigmoid curve with inflection point 0~A/e. Therefore, the Korf-A equation is more suitable to describe tree and stand growth.

The parameter A indicates the final (asymptotic) size of tree growth.

The parameter c is related to initial size (y_0):

$$c = \left(\frac{1}{b}\ln\frac{A}{y_0}\right)^{-(p-1)} \tag{12}$$

When c=0, the growth curve crosses zero point of time axis. Otherwise, the parameter c indicates horizontal displacement on the time axis.

For the parameter b, it gives

$$b = [(p-1)q)]^{\frac{1}{p-1}} \tag{13}$$

As p>1, the parameter b is inversely proportional to the inherent growth rate (q). When p is same, q increases with b decreasing and relative growth rate is higher. So the parameter b reflects growth rate of a tree.

Depending on p, the generalized Korf equation was divided into three growth types (the Korf-A, the Korf-B and the Gompertz function). Therefore, parameter p as power exponent of the attenuate equation decides curve shapes and inflection position. The higher the p value is, the slower the growth is and the later the inflection point appears.

The characteristic values concerned with tree growth for the Korf-A equation are:

1) Age (t_{CAI}) and value (CAI_{max}) of the maximum current annual increment (CAI)

The CAI function of the Korf-A equation is:

$$CAI = \frac{dy}{dt} = y\frac{b}{p-1}(t+c)^{-\frac{p}{p-1}} \tag{14}$$

Solving the equation, $\frac{dCAI}{dt} = \frac{d^2y}{dt^2} = 0$, we get

$$t_{CAI} = t_I = \left(\frac{b}{p}\right)^{p-1} - c \tag{15}$$

Substituting (15) into (14), we can obtain the maximum value of CAI:

$$CAI_{\max} = A\left(\frac{b}{p-1}\right)^{-(p-1)}\left(\frac{p-1}{p}\right)^{-p} e^{-p} \tag{16}$$

2) Age (t_{MAI}) and value (MAI_{max}) of the maximum mean annual increment (MAI)

The MAI function of the Korf-A function is:

$$MAI = \frac{y}{t} = \frac{A}{t}\exp\left(-b(t+c)^{-\frac{1}{p-1}}\right) \tag{17}$$

The age (t_{MAI}) of the maximum MAI is resolving from equation, $\frac{dMAI}{dt} = 0$. The equation may be simplified as:

$$\frac{b}{p-1}t(t+c)^{-\frac{p}{p-1}} - 1 = 0 \tag{18}$$

t_{MAI} can be solved from (18) by iteration approach and the MAI_{max} can be obtained by substituting t_{MAI} into (17). As a special case of the Korf-A equation, when c=0 it gives:

$$t_{MAI} = \left(\frac{b}{p-1}\right)^{p-1} \tag{19}$$

$$MAI_{max} = A(\frac{b}{p-1})^{p-1}e^{-(p-1)} \quad (20)$$

The following is several special solutions of the Korf-A equation:

1) Height or volume growth equation

In this case, equation should be satisfied with initial condition of $y(t) = 0$ when $t = 0$. Let $k = 1/(p-1)$, then from equation (9) we get

$$y = A\exp(-bt^{-k}) \quad A, b>0 \text{ and } k>0 \quad (21)$$

Equation (21) is original Korf equation (Korf, 1939) which is a special case of the Korf-A equation.

2) Diameter and basal area growth equation

With initial condition $y(t) = 0$ when $t = t_0$ and letting $k=1/(p-1)$, from equation (9) it gives

$$y = A\exp(-b(t-t_0)^{-k}) \quad A, b>0 \text{ and } k>0 \quad (22)$$

More generally, with initial condition $y(t_0)=y_0$ and let $k=1/(p-1)$, another special solution can be obtained:

$$y = A\exp(-b(t+c-t_0)^{-k}) \quad A, b>0 \text{ and } k>0 \quad (23)$$

Properties of the Korf-B equation

1) It has asymptotic size: $y \to A$ as $t \to c$.

2) y is an monotonic function related to t. For $0 \le p < 1$, $dy/dt > 0$.

3) It has an inflection point: $t_I = c - (\frac{p}{b})^{1-p}$, $y_I = Ae^{-p}$. The inflection point of the Korf-B equation is $y_I \in [A/e, A]$, because of $0 \le p < 1$. Namely, growth curve belongs to sigmoid that the inflection point is between A/e and A. Depending on the parameter b and p, the growth curve is flexible for $0 \le t \le c$.

4) Age (t_{CAI}) and value (CAI_{max}) of the maximum CAI

$$t_{CAI} = t_I = c - \left(\frac{b}{p}\right)^{1-p} \quad (24)$$

$$CAI_{max} = A\left(\frac{b}{1-p}\right)^{1-p}\left(\frac{p}{1-p}\right)^{p}e^{p} \quad (25)$$

The meanings of parameter A, b and p in the Korf-B equation are same as the Korf-A equation, only that the parameter c is different. It indicates time that finishing whole growth procedure needed under the initial condition, $t=0$ and $y=y_0$. The Korf-B type equation can only describe sigmoid growth curve of biological population that has initial value (y_0).

Application of the Korf-A equation

Self-thinning model

The self-thinning law is always a key problem to forestry researchers. Based on the 3/2 power law (Yoda *et al.* 1963) many models have been developed to describe stand self-thinning process using Gompertz equation (Minowa 1982) and other empirical equations (Smith & Hann 1984; Tang 1994). Based on the Korf-A equation, a new model of self-thinning was developed by Li and Meng (1995). The self-thinning model is

$$N = N_0 - (N_0 - N_{min})\exp(-bDg^{-c}) \quad b>0,\ c \ge 0 \quad (26)$$

Where $b = q/(p-1)$, $c = p-1$, and they are parameters to be estimated.

Growth predicting model

To a tree or a stand with the initial measured data set (t_1, y_1) and remeasured data set (t_2, y_2), it could be assumed that the two points are at the same curve and have the same parameter. The curve to the initially measured data satisfied:

$$y_1 = A\exp(-bt_1^{-k}) \quad (27)$$

Similarly, to the remeasured data also satisfied:

$$y_2 = A\exp(-bt_2^{-k}) \quad (28)$$

From equation (27), parameter b is solved out and is substituted into equation (28). Growth predicting model (difference equation) of individual tree or stand could be got:

$$y_2 = A\left(\frac{y_1}{A}\right)^{\left(\frac{t_1}{t_2}\right)^k} \quad (29)$$

The growth predicting model (29) possess some desirable logical properties of growth projection: 1) As t_2 approaches t_1, y_2 approaches y_1; 2) As $t_2 \to \infty$, y approaches A (upper asymptote y_{max} of tree size); 3) Compatibility can be met. For example, suppose specified value for t_1, t_2 and y_1 are used to predict a future tree size y_2 and a second solution is then obtained to predict another future value y_3 from t_2, t_3

and y_2. The predicted value obtained for y_3 will equal the value obtained in a single equation solution using t_1, t_3 and y_1 as inputs.

Modeling Examples

Height growth of Korean pine plantation

To verify applicability of the Korf-A equation (21) to tree growth, it was fitted to a set of height growth data of 10 stems from Korean pine (*Pinus Koraiensis* Sib.et Zucc.) plantations in Heilongjiang province, northeastern China. We performed a nonlinear regression using STATISTICA (StatSoft Inc 1995) software to find good estimates for the parameters. The estimated values of the parameters, fitting statistics (*df*— degree of freedom, *RSS*—residual sum of squares, $S_{y.x}$ —standard of error estimate and R^2—coefficient of determination for model) and the characteristics to reflect tree growth of the Korf-A growth equation for each tree are presented in Table 3.

Table 3. Estimated parameters and statistics of the Korf-A equation for fitting the height growth of Korean pine plantation

No.of tree	Parameters			Fitted statistics				Characteristics of tree growth			
	A	b	k	df	RSS	$S_{y.x}$	R^2	t_{CAI}	CAI_{max}	t_{MAI}	MAI_{max}
1	32.989	10.026	0.6699	11	0.5801	0.2296	0.9960	7.9873	0.5703	17.172	0.4318
2	32.923	9.924	0.6588	20	1.1863	0.2436	0.9946	8.0198	0.5491	17.290	0.4173
3	34.168	9.079	0.5579	11	0.7323	0.2580	0.9903	8.2767	0.3941	18.322	0.3106
4	34.958	14.187	0.7762	8	0.9214	0.3394	0.9916	10.491	0.6003	21.991	0.4383
5	27.109	11.178	0.7381	7	0.0684	0.0989	0.9990	8.2489	0.5421	17.444	0.4009
6	27.106	16.775	0.8663	10	0.5639	0.2375	0.9947	10.689	0.5489	21.965	0.3891
7	36.464	7.879	0.5580	10	0.1646	0.1283	0.9984	6.4181	0.5425	14.207	0.4276
8	29.346	15.474	0.8063	9	0.3623	0.2006	0.9951	10.988	0.5134	22.878	0.3711
9	31.196	15.745	0.7230	8	0.2668	0.1826	0.9931	13.620	0.3641	28.905	0.2707
10	30.474	11.180	0.6323	8	0.2084	0.1614	0.9953	10.157	0.3705	22.046	0.2843

Note: *A, b, k*- parameters of equation; *df*-degree of freedom; *RSS*-residual sum of squares; $S_{y.x}$-standard of error estimate; R^2-coefficient; t_{CAI}-Age current annual increment; CAI_{max}-the maximum current annual increment; t_{MAI}-Age mean annual increment; MAI_{max}-the maximum mean increment.

The result is shown that R^2 is higher then 0.99 and $S_{y.x}$ is less than 0.4 of which is less than 5% of total height for each tree. Fitness of height growth data is generally good.

Self-thinning model for Sugi (Cryptomeria japonica D. Don) plantation

Using the data of stem number per hectare (*N*) and mean diameter (*Dg*) from five permanent plots of varying spacing for Sugi plantations of experimental forest of National Taiwan University (Yang, 1989), the self-thinning equation (26) was tested by Li and Meng (1995).

Conclusion

The generalized Korf equation is useful because of not only its capability of description and analysis, but also better prediction. The attenuate equation was presented and the generalized Korf growth equation was derived from the self-control model of tree growth. According to different power exponent *p* of the attenuate equation, three types of growth equations: the Korf-A (p>1), the Korf-B (0<p<1) and Gompertz functions (p=1) were developed. The Korf-A equation can be used to describe the growth of trees or stands of that the inflection point is between 0 and *A*/*e*. The Korf-B equation, which the inflection point is between *A*/*e* and *A*, can be applied to fit the growth of biological population (or the seasonal growth of various organs) with an initial value. Between two equations, there is the Gompertz equation to separate each other. All of the three equations are independent for allometric relationship. The Korf-A equation has better properties in describing the growth process of a tree or a stand and we can get some characteristics values to reflect tree growth. Theses important properties are useful in actual applications. The applications of the Korf-A equation to modeling height growth and stand self-thinning showed that fitness of tree growth and stand development were generally good.

References

Brewer, J.A., Burns, P.Y., Cao, Q. V. 1985. Short-term projection accuracy of five asymptotic height-age curves for loblolly pine [J]. For. Sci., **31**: 414-418.

Li Fengri. 1987. Study on diameter distribution and yields models for natural dahurian larch stands [J]. J. Northeast For. Univ., **15**(4): 8-16(In Chinese).

Li Fengri. 1995. A simulation system of stand dynamics for larch plantation [D]. Ph.D. dissertation. Beijing Forestry Univ., p203 (In Chinese).

Li Fengri, Meng Lingbin. 1995. Maximum density line and model of self-thinning for plantation [J]. J. Northeast Forestry University (English Edition), **6**(4):1-7.

Lundqvist, B. 1957. On the height growth in cultivated stands of pine and spruce in Northern Sweden [J]. Medd. Fran Statens Skogforsk., band **47**(2): 64.

Minowa, M. 1982. A theoretical approach to forest growth model (I) The log-Mitscherlich theory [J]. J. Jap. For. Soc., **64**(12): 461-467 (In Japanese).

Osumi, S., Ishikawa,Y. 1983. Applicability of the Richards' growth function to analysis of growth of tree [J]. Scientific Reports of the Kyoto Prefecture Univ. Agriculture, **35**: 49-76 (In Japanese).

Pienaar,L.V., Turnbull, K.J. 1973. The Chapman-Richards generalization of Von Bertalanffy's growth model for basal area growth and yield in even-aged stand [J]. For. Sci., **19**(1): 2-22.

Chapman-Richards, F.J. 1959. A flexible growth function for empirical use [J]. J. Exp. Bot., **10**: 290-300.

Smith ,N.J., Hann, D.W. 1984. A growth model based on the self-thinning rule [J]. Can. J. For. Res., **16**: 330-334

Stage, A.R. 1963. A mathematical approach to polymorphic site index curves for grand fir [J]. For. Sci., **9**(2): 167-180.

StatSoft, Inc. 1995. STATISTICA for Windows [C] [Computer program manual]. Tulsa. OK..

Tang, S.Z, Meng, C.H., Meng, F.R., *et al.* 1994. A growth and self-thinning model for pure even-aged stands: theory and application [J]. For. Ecol. Manage, **70**: 67-73.

Yang, Y.C. 1989. Studies on the relationship of structure, allometric growth of tree and self-thinning rule based on the principles of plant ecology [J]. Q. J. Exp. Forest. National Taiwan University, **4**(1): 147-163 (in Chinese).

Yoda, K., Kira, T., Ogawa, H., *et al.* 1963. Self-thinning in overcrowed pure stands under cultivated and natural conditions [J]. J. Biol. Osaka City Univ., **14**: 107-109.

Zarnovican, R. 1979. Korf's growth equation [J]. For. Chron., **55**: 194-197 (In French).

Zeide, B. 1989. Accuracy of equations describing diameter growth [J]. Can. J. For. Res., **19**: 1283-1286.

Zeide, B. 1993. Analysis of growth equations [J]. For. Sci., **39**(3): 594-616.